



CARIBBEAN EXAMINATIONS COUNCIL
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS
MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE sections. Answer ALL questions.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
6. **If you use the extra page(s), you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**
7. You may use a silent, non-programmable calculator.

Examination Materials:

Mathematical formulae and tables (Revised 2012)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) A farmer, in the business of growing fodder for livestock, has 50 acres available for planting alfalfa and corn. The cost of alfalfa seeds is \$60 per acre, and the cost of corn seeds is \$45 per acre. The total cost of labour will amount to \$40 per acre for alfalfa and \$80 per acre for corn. The farmer does not wish to spend more than \$2700 for seeds and \$3200 for labour. The income per acre from alfalfa is \$110 and that from corn is \$150.

(i) Using x to represent the number of acres planted with alfalfa and y to represent the number of acres planted with corn, formulate a linear programming problem for the information given. State clearly the maximizing function.

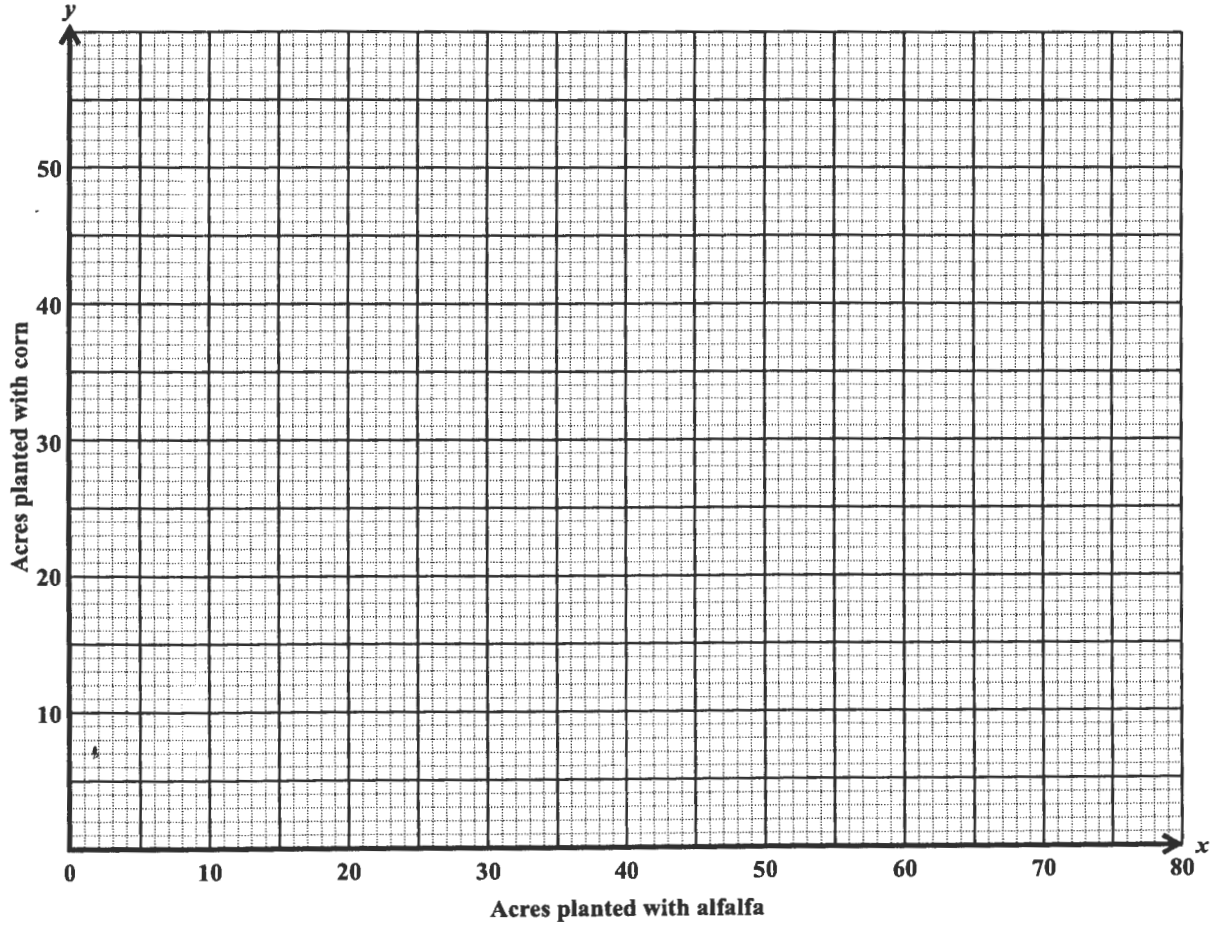
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[5 marks]

(ii) On the grid provided on page 5, draw all the lines defined by the constraints and shade the feasible region which satisfies ALL of these constraints. [8 marks]



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- (iii) Determine the number of acres of alfalfa and the number of acres of corn that the farmer must plant to maximize his income.

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[6 marks]

- (b) Write the contrapositive of $p \rightarrow \sim q$.

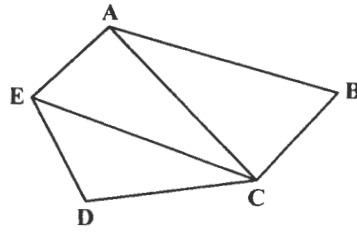
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[2 marks]



(c) Given the following diagram, list



- the vertices that have a degree greater than 2

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[2 marks]

- all the paths from *C* to *E*.

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[2 marks]

Total 25 marks

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2. (a) A project consisting of 9 activities, *R, S, T, U, V, W, X, Y, Z*, is given in the table below. The duration times and immediate predecessors of the activities are also shown.

Activity	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
Duration Time in Minutes	8	10	12	9	6	7	5	6	4
Immediate Predecessors	–	<i>R</i>	–	<i>S</i>	<i>T, S</i>	<i>U, V</i>	<i>W</i>	<i>W</i>	<i>X, Y</i>

- (i) In the space below, construct the activity network for the project.

[6 marks]



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(ii) Complete the following table.

Activity	Earliest Start Time	Latest Start Time	Float
<i>R</i>			
<i>S</i>			
<i>T</i>			
<i>U</i>			
<i>V</i>			
<i>W</i>			
<i>X</i>			
<i>Y</i>			
<i>Z</i>			

[4 marks]

(iii) State the critical path of the activity network.

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[2 marks]

(iv) Determine the MINIMUM time needed to complete the project.

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[2 marks]

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- (b) The following table gives the daily income, in dollars, earned by four workers *A*, *B*, *C* and *D* to perform the tasks *R*, *S*, *T* and *U*. All of the workers are assigned and each worker is given a different task.

Worker	Task			
	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>
<i>A</i>	232	310	250	282
<i>B</i>	305	305	330	294
<i>C</i>	209	250	285	255
<i>D</i>	357	295	307	305

- (i) Use a Hungarian algorithm to determine the task to which EACH worker must be assigned in order to MAXIMIZE the total income.

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[9 marks]

(ii) Determine the TOTAL income of the workers for the four tasks.

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[2 marks]

Total 25 marks
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SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) Four persons are to be chosen to sit on a family life committee, from a group which consists of 6 males and 4 females. Three of the males are local. Find the probability that the 3 locals are on the committee.

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[5 marks]

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(b) A continuous random variable, X , with probability density function $f(x)$ given by $f(x) = k$, where k is a constant, assumes values between $x = 1$ and $x = 5$.

(i) Calculate the value of the constant k and sketch the graph of $f(x)$ in the space provided below.

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[4 marks]

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(ii) Determine $E(X)$ and $\text{Var}(X)$.

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[6 marks]

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(iii) Calculate $P(3 < X < 4.5)$.

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[3 marks]



- (iv) Determine the cumulative distribution function, $F(x) = P(X \leq x)$, and sketch its graph in the space provided below.

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(v) Calculate the median, m , of x .

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[3 marks]

Total 25 marks



4. (a) Rectangular metal sheets have an average of 2 blisters for every 15 m² of metal. Assuming that these blisters may be modelled by a Poisson distribution, determine, to **3 significant figures**, the probability that a piece of metal 15 m² will contain **at most** three blisters.

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[4 marks]

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- (b) The probability that a customer who goes to the Land Tax Department intends to pay arrears is 0.003 and is independent of all other customers. On a particular day, 500 customers go to the Land Tax Department to pay their taxes. Justifying the use of a suitable approximation, calculate the probability that **at least 2** customers intend to pay their arrears.

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(c) The number of calls received at an office is found to follow a Poisson distribution with mean 35.

(i) State clearly the distribution which can be used to approximate the Poisson distribution.

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[1 mark]

(ii) Calculate the probability that in one hour less than 32 calls are received.

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[4 marks]

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(iii) Calculate the probability that in one hour between 31 and 39 calls are received.

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[4 marks]

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- (d) The table below gives the preference of 250 persons who were asked to choose among five flavours of a certain tea.

	Flavour					Total
	Original	Cranberry and Blueberry	Blueberry and Mango	Apple and Pomegranate	Apple and Spice	
[O] Number Preferring Brand	60	50	40	45	55	
[E] Expected Number Preferring Brand						

- (i) Complete the table by giving the expected number of persons preferring EACH brand, assuming that each brand is equally likely to be preferred. [1 mark]
- (ii) Perform a χ^2 goodness-of-fit test at the 1% significance level to determine whether a uniform distribution fits the tea-drinking preference of the 250 persons.

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[6 marks]

Total 25 marks

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SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

[Take $g = 10 \text{ ms}^{-2}$]

5. (a) Two particles, A and B , of masses $(m + 3)$ kg and m kg respectively are connected by a light inextensible string which passes over a fixed, smooth, light pulley located at the edge of a table. The system is initially held at rest with B hanging freely at the end of the string that hangs over the edge of the table, while A rests on the rough surface of the table. The coefficient of friction between Particle A and the surface is 0.36. The acceleration of the system is 2.5 ms^{-2} when the system is released.
- (i) Draw a diagram to illustrate this information, showing the forces acting on the system.

[4 marks]



(ii) Determine the masses of the particles, A and B .

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[6 marks]

(iii) Calculate the tension, T , in the string.

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[2 marks]

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(b) An object, on the edge of a cliff that is 45 m above the ground, is projected with a speed of 20 ms^{-1} at an angle of 30° with respect to the horizontal.

(i) If air resistance is ignored, determine the time taken for the object to hit the ground.

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[9 marks]

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6. (a) A 10 kN load is pulled steadily up a track inclined at 25° to the horizontal by a horizontal force, P . If the coefficient of friction, μ , is 0.2, determine the

(i) horizontal force P

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[7 marks]

(ii) frictional force.

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(b) A truck provides an impulse of 420 N to set in motion a stationary trolley of mass 45 kg which then moves freely at velocity, V_1 , against a track resistance of 0.8 Nkg^{-1} . If the trolley reaches a velocity, V_2 , after 5 seconds, calculate the velocities V_1 and V_2 .

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[10 marks]



- (c) Two ropes of equal length attached to an object of mass 80 kg are used to pull the object out of a hole. If the ropes are 60° apart, determine the tensions in the ropes.

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[5 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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